Topology Project

Topology and Geometry of Low-dimensional Manifolds 2023

November 1 (Wed) – November 3 (Fri), 2023

Face-to-face

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Schedule

JST	1 (Wed)	2 (Thu)	3 (Fri)
10:00–11:00	Nakanishi & Negami*	Taniguchi (1)	Kydonakis
11:30–12:30	Tanaka	Taniguchi (2)	Watanabe
14:30–15:30	Koda	Ohshika	
16:00–17:00	Teragaito	Paoluzzi	

*10:00–10:30 and 10:40–11:10

Abstract

November 1 (Wed)

Toshihiro Nakanishi (Shimane University)

Title. Fundamental regions for a Kleinian group

Abstract. A Kleinian group admits fundamental polyhedra and Ford and Dirichlet polyhedra are well known among them. In this talk I present matrix representations of genus 2 surface group and report what I observed about those "fundamental polyhedra" for a subgroup of $SL(2, C)$.

Haru Negami (Chiba University)

Title. Generalization of the construction method of the representation of braid groups

Abstract. The Long-Moody construction is a method for obtaining a representation of the braid group from a given representation of the semidirect product of the free group F_n and the braid group B_n defined by Artin representation. The braid group is defined as the mapping class group of a closed disk with n holes, $D\backslash\{n\text{-points}\}$, and the fundamental group of $D\backslash\{n\text{-points}\}$ is isomorphic to F_n. Thus, Artin representation has a geometric realization such as the transformation of the loops on $D\backslash\{n\text{-points}\}$. Wada generalized the Artin representation and obtained the group invariants of links.

In this talk, we first introduce the extension of the method to obtain an infinite series of representations of the braid group by using the convolution of Dettweiler-Reiter, defined by an integral transform. In particular, this construction has a correspondence with the middle convolution of KZ-type equations in the case of pure braid groups, giving an irreducible representation. Second, we further extend the method to one of the Wada types of semidirect product. We then discuss the connection between this construction and the unitarity of the representation.

reference K. Hiroe and H. Negami. Long-moody construction of braid representations and Katz middle convolution. arXiv preprint arXiv:2303.05770, 2023.

Toshifumi Tanaka (Gifu University)

Title. On the symmetry of ribbon knots

Abstract. A ribbon knot is a knot which bounds an immersed disk, in the 3-space, with only ribbon singularities. For example, the connected sum of a knot and its mirror image with orientation reversed is a ribbon knot. This construction can be generalized as a symmetric union which was introduced by Kinoshita and Terasaka in 1957. Moreover, Lamm generalized the construction in 2000 and interestingly gave a question which asks whether every ribbon knot is a symmetric union. In this talk, we will discuss the question and will also discuss the amphicheirality of symmetric unions.

Yuya Koda (Keio University)

Title. Shadows, divides and hyperbolic volumes

Abstract. A divide is the image of a proper and generic immersion of a compact 1-manifold into the 2-disk. Due to A'Campo's theory, each divide is associated with a link in the 3-sphere. In this talk, we reveal a hidden hyperbolic structure in the theory of links of divides. More precisely, we show that the complement of the link of a divide can be obtained by Dehn filling a hyperbolic 3-manifold that admits a decomposition into several ideal regular tetrahedra, octahedra and cuboctahedra, where the number of each of those three polyhedra is determined by types of the double points of the divide. This immediately gives an upper bound of the hyperbolic volume of the links of divides, which is shown to be asymptotically sharp. An idea from the theory of Turaev's shadows plays an important role here. This talk is based on joint work with Ryoga Furutani (Hiroshima University).

Masakazu Teragaito (Hiroshima University)

Title. Chiral hyperbolic knots with vanishing Upsilon invariants

Abstract. For a knot in the 3-sphere, its Upsilon invariant is a continuous, piecewise linear function defined on the interval $[0, 2]$. It is a concordance invariant, so vanishes for slice knots. Since it only changes the sign for the mirror image with reversed orientation, it also vanishes for amphicheiral knots. For alternating knots, more generally, quasi-alternating knots, the Upsilon invariant is determined only be the signature. In particular, it vanishes if such a knot has zero signature. On the other hand, it is known

that the Upsilon invariant is a non-zero convex function for L-space knots. In this talk, we construct an infinite family of hyperbolic knots, each of which has zero Upsilon invariant, but is chiral, non-slice, non-alternating. To confirm that the Upsilon invariant vanishes, we calculate the full knot Floer complex.

November 2 (Thu)

Masaki Taniguchi (Kyoto University)

Title. Instanton Floer homology and its applications I

Abstract. Floer theory has made significant contributions to the development of 4-dimensional topology. The theory extracts subtle information about differential structures by studying the solution spaces of differential equations defined on 4-dimensional manifolds. Among the various branches of Floer theory, instanton Floer theory stands out for its deep connection to the fundamental groups of 3 and 4-dimensional manifolds, leading to numerous applications in topology. In this presentation, we provide an introduction to instanton Floer theory.

Title. Instanton Floer homology and its applications II

Abstract. Instanton Floer theory is defined as a model for infinite-dimensional Morse homology. Within its construction, the analog of a Morse function is referred to as the Chern-Simons functional, and its critical values become topological invariants. By incorporating information from these critical values into the theory, instanton Floer theory exhibits intriguing applications not found in other Floer theories. This presentation focuses on highlighting such applications and introduces recent research developments in this area.

Ken'ichi Ohshika (Gakushuin University)

Title. The broken windows only theorem and the bounded image theorem

Abstract. Both the broken windows only theorem and the bounded image theorem constitute steps of Thurston's original proof of the unifomisation theorem for Haken manifolds. Looking at his plan of the entire proof, we realise that his proof for the bounded image theorem realies on the broken windows only theorem, in particular its second statement. We show that this second statement is in fact false, giving counter examples. Nevertheless we can prove that the bounded image theorem itself is valid, making use of recent tools of Kleinian group theorem. The latter part is joint work with Cyril Lecuire (ENS Lyon).

Luisa Paoluzzi (Aix-Marseille Université)

Title. Relatively hyperbolic groups with planar boundaries

Abstract. The Bowditch boundary of a group G relatively hyperbolic with respect to a family P of subgroups is a compactum on which G acts and that is an invariant of the pair

(G, P). Groups of hyperbolic knots with respect to their peripheral subgroups provide the most basic examples of relatively hyperbolic group pairs. A natural and very general question in this context is the following: given a compactum K and assuming that it is the Bowditch boundary of some pair (G, P), what can we deduce about G and P from K? I will present a joint work with Peter Haïssinsky and Genevieve Walsh, in which we address this question under the hypothesis that K is planar, i.e. embeds into the 2-sphere, and the action of G extends to the whole sphere. We are specifically interested in the case where K is a Schottky set. We show that Schottky sets come in three combinatorial flavours for two of which it is possible to give a description of the structure of G.

November 3 (Fri)

Georgios Kydonakis (Universität Heidelberg)

Title. Fock bundles and Teichmüller spaces

Abstract. Higher Teichmüller theory is concerned with the study of special connected components of character varieties sharing analogous properties to the classical Teichmüller space. Fixing a complex structure on the underlying topological surface introduces powerful holomorphic techniques through certain holomorphic pairs called Higgs bundles, which correspond to fundamental group representations via the non-abelian Hodge correspondence. Yet, a rather adverse aspect of the correspondence is that is fails to transfer the action of the mapping class group on character varieties to the moduli space of Higgs bundles. We will introduce a similar class of augmented bundles over a topological surface that we call Fock bundles which does not require fixing any complex structure on the underlying surface. We conjecture that there is an alternative passage to the one given by the non-abelian Hodge correspondence from such pairs to certain higher rank Teichmüller spaces that is independent of the complex structure on the surface. This is joint work with Charles Reid (Austin) and Alexander Thomas (Heidelberg).

Tadayuki Watanabe (Kyoto University)

Title. Smooth bundles of disks from Kontsevich's graph complex with higher valences

Abstract. Ideas in low-dimensional topology are often powerful also for higher dimensional manifold bundles. For example, Goussarov-Habiro's theory of surgery on 1,3-valent graphs in 3-manifolds can be generalized for higher dimensional manifold bundles. Recently, we further generalized the 3-valent graph surgery of bundles to graphs with valences at most 5 to obtain a chain map from the corresponding part in Kontsevich's graph complex to the singular chain complex of $B\mathrm{Diff}_\partial(D^{2k})$. In particular, a nontrivial graph cocycle including the pentagon wheel (with one 5-valent vertex) is then mapped to an $(8k-10)$-cycle in $B\mathrm{Diff}_\partial(D^{2k})$. We have a crucial evidence that this cycle is nontrivial in $H_{8k-10}(B\mathrm{Diff}_\partial(D^{2k}); \mathbb{Q})$. This is a joint work in progress with Boris Botvinnik.