Topology Project

Topology and Geometry of Low-dimensional Manifolds

October 29 (Mon) - November 1 (Thu), 2018

Nara Women's University, Collaboration Center Z306

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Schedule

	29 (Mon)	30 (Tue)	31 (Wed)	1 (Thu)
10:00-11:00		Palesi (2)	Kato	Saito
11:30-12:30		Morita	Kabaya	Heusener
13:00-14:00	Palesi (1)		(Lunch)	
14:30-15:30	Ishikawa		Akiyoshi	
16:00-17:00	Yuasa		Fillastre	

Abstract

Hirotaka Akiyoshi (Osaka City University)

Title. From one-cone tori to two-bridge cone manifolds

Abstract. In his unfinished paper, Jørgensen characterized the combinatorial structures of the Ford domains of quasifuchsian punctured torus groups. The work was extended by Akiyoshi-Sakuma-Wada-Yamashita to certain cone hyperbolic manifolds which lie in the outside of the space of puntured torus groups to give paths from the four-times punctured sphere to two-bridge knot complements. In this talk, I will explain an ongoing project which aims to replace the "puncture" to "cone singularity". In particular, I will present numerical observations on paths from four-cone spheres to figure-eight cone manifolds.

François Fillastre (Université de Cergy-Pontoise)

Title. Co-Minkowski space and hyperbolic surfaces

Abstract. There are many ways to parametrize two copies of Teichmueller space by constant curvature -1 Riemannian or Lorentzian 3d manifolds (for example the Bers double uniformization theorem). We present the co-Minkowski space (or half-pipe space), which is a constant curvature -1 degenerated 3d space, and which is related to the tangent space of Teichmueller space. As an illustration, we give a new proof of a theorem of Thurston saying that, once the space of measured geodesic laminations on a compact hyperbolic surface is identified with the tangent space of Teichmueller space via infinitesimal earthquake, then the length of laminations is an asymmetric norm. Joint work with Thierry Barbot.

Michael Heusener (Université Clermont Auvergne)

Title. Computing the scheme of characters in $\mathrm{SL}(2,\mathbb{C})$

Abstract. This is joint work with Joan Porti. The aim of this talk is to compute the character scheme of a finitely presented group Γ. Let

$$\Gamma \cong \langle \gamma_1, \dots, \gamma_n \mid r_1, \dots, r_l \rangle$$

be a presentation of Γ. The natural surjection from the free group on n generators is denoted by

$$\pi \colon \mathbb{F}_n \twoheadrightarrow \Gamma$$

The scheme of representations and characters in $\mathrm{SL}(2,\mathbb{C})$ are denoted respectively by $R(\Gamma, \mathrm{SL}(2,\mathbb{C}))$ and $X(\Gamma, \mathrm{SL}(2,\mathbb{C}))$. The corresponding rings or functions are the universal algebra of $\mathrm{SL}(2,\mathbb{C})$-representations

$$A(\Gamma) = \mathbb{C}[R(\Gamma, \mathrm{SL}(2,\mathbb{C}))]$$

and the universal algebra of $\mathrm{SL}(2,\mathbb{C})$-characters

$$B(\Gamma) = \mathbb{C}[X(\Gamma, \mathrm{SL}(2,\mathbb{C}))] \cong A^{\mathrm{SL}(2,\mathbb{C})}.$$

The reduction of $B(\Gamma)$ is precisely the algebra of functions of $X(\Gamma, \mathrm{SL}(2,\mathbb{C}))$ as a variety, but a priory this algebra does not need to be reduced.

For $\gamma \in \mathbb{F}^n$, denote by $t_\gamma \in B(\mathbb{F}^n)$ the evaluation function at this element. Montesinos-Amibilia and González-Acuña have shown in that for the reduction of $B(\Gamma)$ holds

$$B(\Gamma)^{\mathrm{red}} \cong B(\mathbb{F}^n)/I^{\mathrm{rad}}$$

where

$$I = (\langle t_{r_s} - 2,\ t_{\gamma_i r_s} - t_{\gamma_i} \mid 1 \le s \le l,\ 1 \le i \le n \rangle)\ .$$

Explicit computations show that in general $B(\Gamma) \ncong B(\mathbb{F}^n)/I$.

The aim of this talk is to compute the algebra $B(\Gamma)$:

Theorem 1. *Let $I_\Gamma < B(\mathbb{F}^n)$ denote the ideal.*

$$I_\Gamma = \langle t_{r_s} - 2,\ t_{\gamma_i r_s} - t_{\gamma_i},\ t_{\gamma_j\gamma_k r_s} - t_{\gamma_j\gamma_k} \mid 1 \le s \le l,\ 1 \le i \le n,\ 1 \le j < k \le l \rangle.$$

Then $B(\Gamma) \cong B(\mathbb{F}^n)/I_\Gamma$.

Various examples and applications will be given.

Katsumi Ishikawa (RIMS, Kyoto University)

Title. On zeros of the Alexander polynomials of alternating knots

Abstract. In 2002, Hoste proposed a conjecture based on his computer experiments: the real part of every root of the Alexander polynomial of an alternating knot will be greater than -1. In fact, this is true for many knots with small crossing numbers, and the speaker showed that it holds for the 2-bridge knots.

In this talk, however, we prove that there exist infinitely many counterexamples for Hoste's conjecture, which are Montesinos knots, and show some of them explicitly. Then, it is natural to ask whether there exists a lower bound for the real parts of the zeros, and we also give a lower bound in the case of the alternating Montesinos knots.

This talk is partly based on a joint work with Mikami Hirasawa and Masaaki Suzuki.

Yuichi Kabaya (Kitami Institute of Technology)

Title. Deformation of ideal octahedra and quasi-Fuchsian once-punctured torus groups

Abstract. For a once-punctured torus S, the (type-preserving) $\mathrm{PSL}(2,\mathbb{C})$-character variety is the set of (conjugacy classes of) $\mathrm{PSL}(2,\mathbb{C})$-representations of the fundamental group of S preserving parabolics. Fixing the (complex) length of a simple closed curve on S, we obtain a 1-dimensional subvariety, which is called the linear slice. In this talk, we study the subset consisting of discrete faithful representations in the linear slice, especially from the view point of deformation of ideal hyperbolic octahedra.

Motoko Kato (University of Tokyo)

Title. Group actions on finite dimensional non-positively curved spaces

Abstract. We give a condition for group elements to have fixed points, whenever groups act on finite dimensional non-positively curved spaces. As an application, we show that Thompson's groups T, V and their generalizations have property $\mathrm{F}\mathcal{A}_k$ for every natural number k.

Shigeyuki Morita (The University of Tokyo/Tokyo Institute of Technology)

Title. Motivic Lie algebra and cohomology of moduli spaces of graphs and curves

Abstract. The motivic Lie algebra which is a free Lie algebra generated by the Soulé elements plays fundamental roles in number theory as well as many related branches of mathematics. In topology, it first appeared in the Johnson cokernel through works of Ihara, Deligne, Oda, Nakamura, Matsumoto and finally Brown. Recently, Chan, Galatius and Payne proved that it also appears in the totality of $(4g-6)$-dimensional cohomology of the moduli space of curves of genus g. However, both the precise description of the former and topological meaning of the latter remain quite mysterious.

In this talk, we first survey the above situation. Then we present a conjectural relationship between the motivic Lie algebra and the totality of unstable $4n$-dimensional cohomology class of the moduli space of graphs of rank $2n+2$, called the Morita classes, based on a joint work with Sakasai and Suzuki.

Frederic Palesi (Aix-Marseille Université)

Title Character varieties of surface groups and trace reduction

Abstract. In this talk, we will investigate several questions about the simple trace spectrum of a representation of a surface group into PSL(2,R) or PSL(2,C). This can be thought as a generalisation of the simple length spectrum of an hyperbolic surface, in a more algebraic context. The behaviour of this set, in particular the value of the systole and the growth of the spectrum, gives important informations about the dynamic of the action of the mapping class group on the character variety. In many cases of low complexity there is a simple combinatorial model that can help us understand this spectrum.

The goal of these lectures is to explain these combinatorial models in different cases, and how we can use arguments of trace reduction to find a set of curves with small traces. This helps us construct large domain of discontinuity for the action of the Mapping Class Group. If time permits, we will see how to apply these results to answer questions of Bowditch about type-preserving representations, of Goldman about ergodicity of mapping class group actions on certain components of the character variety, and also to get a new way to produce systolic inequalities for certain hyperbolic surfaces.

Toshio Saito (Joetsu University of Education)

Title. Tunnel number of knots and generalized tangles

Abstract. It is known that there is a fundamental inequality related to tunnel number of a knot and tangles obtained by its tangle decomposition. The inequality gives an upper bound for the tunnel number of the knot, and there exists a knot in the 3-sphere so that the inequality is non-strict.

This talk will discuss a slight generalization of those. More precisely, we will consider generalized tangles which are pairs of a handlebody and a disjoint union of arcs properly embedded in the handlebody.

Wataru Yuasa (Kyoto University)

Title. On power subgroups of Dehn twists in hyperelliptic mapping class groups

Abstract. In this talk, we would like to discuss indices of power subgroups in the mapping class group of a punctured sphere and ones in the hyperelliptic mapping class group of an oriented closed surface. The main tool we use is a projective representation of the mapping class group obtained through the linear skein theory. Our works are the study of "the remaining case" of Masbaum's work and a generalization of Stylianakis' work.