

**Topology Project**  
**Topology and Geometry of Low-dimensional Manifolds**

October 24 (Tue) - October 27 (Fri), 2017

Nara Women's University, Collaboration Center Z306, Z103

Supported by JSPS KAKENHI Grant Number 15H05739, 16H02145, 16K05161, 17H02843

**Schedule**

	24 (Tue)	25 (Wed)	26 (Thu)	27 (Fri)
10:00-11:00		Tsuji	Yasuhara*	Kuno
11:30-12:30		Lescop I	Lescop II*	Paoluzzi
13:00-14:00	Papadopoulos I		(Lunch)	
14:30-15:30	Papadopoulos II		Ishibashi	
16:00-17:00	Morita		Oba	

\*The morning session on Oct. 26 will be held at Room Z103. The other will be held at Z306.

**Abstract**

**Tsukasa Ishibashi (The University of Tokyo)**

**Title.** On a Nielsen-Thurston classification theory on cluster modular groups

**Abstract.** It is known that each element of the mapping class group of an orientable surface is classified into three types. These types are characterized by fixed point properties of a natural action on a closed disk, which is the Thurston compactification of the Teichmuller space. These are the Nielsen-Thurston classification theory. On the other hand, by Fock-Goncharov, the mapping class group and the Teichmuller space are generalized to cluster modular groups and cluster ensembles respectively. For particular choices of the input data, these concepts can describe higher Teichmuller spaces and the mapping class group action on them in a combinatorial language. In this talk, I will give a classification of elements of the cluster modular group, which is an analogue of the Nielsen-Thurston classification. Then they are related with fixed point properties of the action on the tropical compactification of the cluster ensemble.

**Yusuke Kuno (Tsuda University)**

**Title.** Formality of the Goldman-Turaev Lie bialgebra and the Kashiwara-Vergne problem in positive genus

**Abstract.** This talk is based on a joint work with A. Alekseev, N. Kawazumi and F. Naef. Given a compact oriented surface with non-empty boundary and a framing of the surface,

one can define the Lie bracket (Goldman bracket) and the Lie cobracket (Turaev bracket) on the vector space spanned by free homotopy classes of loops on the surface. These maps are of degree minus two with respect to a certain filtration. Then one can ask the formality of this Lie bialgebra: is the Goldman-Turaev Lie bialgebra isomorphic to its associated graded? For surfaces of genus zero, we showed that this question is closely related to the Kashiwara-Vergne (KV) problem in Lie theory (arXiv:1703.05813). A similar result was obtained by G. Massuyeau by using the Kontsevich integral. Our new topological interpretation of the classical KV problem leads us to introduce a generalization of the KV problem in connection with the formality of the Goldman-Turaev Lie bialgebra for surfaces of positive genus. We will discuss the existence and uniqueness of solutions to the generalized KV problem.

**Christine Lescop (Institut Fourier, Université Grenoble Alpes, CNRS)**

**Title I.** Counting Feynman diagrams in 3-manifolds

**Abstract.** We will discuss ways of counting some Feynman diagrams in 3-manifolds in order to produce topological invariants for 3-manifolds and for their links, following Gauss, Witten, Bar-Natan, Kontsevich and others. This introductory talk will focus on the simplest two invariants that can be defined in this setting. These two invariants are the linking number of 2-component links and the Casson invariant of integer homology 3-spheres. We will see how such an interpretation of the Casson invariant leads to a formula for the Casson invariant in terms of Heegaard diagrams.

**Title II.** On tangle invariants that count Feynman diagrams

**Abstract.** In our second talk, we will show how the tools introduced in the first talk lead to more general invariants of tangles in rational homology cylinders. We will describe a functor on the category of these tangles. Our functor counts Feynman graphs in the sense of the previous talk. It generalizes both a universal Vassiliev link invariant and a universal finite type invariant of rational homology 3-spheres. We will illustrate our constructions with the example of a coefficient of the Alexander polynomial of null-homologous knots in rational homology 3-spheres, which can be explicitly extracted from this functor.

**Shigeyuki Morita (The University of Tokyo/Tokyo Institute of Technology)**

**Title.** Characteristic classes of flat bundles and invariants of homology spheres

**Abstract.** First we discuss the critical monodromy as well as the degrees of the non-triviality of characteristic classes of flat vector bundles, such as the Euler class and the Borel classes. Then we relate this to a conjectural geometric meaning of the Morita classes which are certain unstable characteristic classes of automorphism groups of free groups. Finally, we present our dream to obtain a series of homology cobordism invariants for homology spheres by extending the above picture. This talk is based on a joint project with Takuya Sakasai and Masaaki Suzuki.

**Takahiro Oba (Tokyo Institute of Technology)**

**Title.** Contact manifolds with infinitely many Stein fillings from low to high dimensions

**Abstract.** A Stein filling of a given contact manifold is a Stein domain whose boundary is contactomorphic to the given contact manifold. Open books, Lefschetz fibrations, and mapping class groups of their fibers in particular help us to produce various contact manifolds and their Stein fillings. However, little is known about mapping class groups of higher-dimensional manifolds. This is one of the reasons that it was unknown whether there is a contact manifold of dimension  $> 3$  with infinitely many Stein fillings. In this talk, after reviewing the 3-dimensional case, I will construct higher-dimensional contact manifolds with infinitely many Stein fillings by using open books and Lefschetz fibrations.

**Luisa Paoluzzi (Aix-Marseille University)**

**Title.** Non equivalent coverings between fibred hyperbolic manifolds

**Abstract.** I will present an elementary construction of surface coverings that allows to find pairs of conjugate pseudo-Anosov maps that lift to the same pseudo-Anosov via two non equivalent coverings. As a consequence, it is possible to produce pairs of hyperbolic 3-manifolds which fibre over the circle so that the first manifold of the pair covers the second in two non equivalent ways, with both coverings preserving the fibration.

The same techniques provide examples of non-isometric hyperbolic 3-orbifolds with the same topological type and volume.

This is joint work with J. Los (Marseille) and A. Salgueiro (Coimbra).

**Athanase Papadopoulos (IRMA, Université de Strasbourg, CNRS)**

**Title.** Teichmüller spaces of surfaces with boundary (I and II)

**Abstract.** I will describe some recent work on the Teichmüller spaces of surfaces with boundary, and in particular concerning the arc metric on this space.

**Shunsuke Tsuji (The University of Tokyo)**

**Title.** A formula for the action of Dehn twists on the HOMFLY-PT type skein algebra and its application

**Abstract.** We give an explicit formula for the action of the Dehn twist along a simple closed curve of a surface on the completed HOMFLY-PT type skein modules of the surface in terms of the action of the completed HOMFLY-PT type skein algebra of the surface. As an application, using this formula, we construct an invariant for an integral homology 3-sphere which is an element of  $\mathbb{Q}[\rho][[h]]$ .

**Akira Yasuhara (Tsuda University)**

**Title.** Arrow calculus for welded links

**Abstract.** We develop a diagrammatic calculus for welded knotted objects. We define Arrow presentations, which are essentially equivalent to Gauss diagrams but carry no sign on arrows, and more generally w-tree presentations, which can be seen as ‘higher order Gauss diagrams’. We provide a complete set of moves for Arrow and w-tree presentations. This Arrow calculus is used to characterize finite type invariants of welded knots and long knots.

This is a joint work with Jean-Baptiste Meilhan (University of Grenoble Alpes).