**Topology Project**

# Topology and Geometry of Low-dimensional Manifolds

June 5 (Wed) - June 8 (Sat), 2019

Shiinoki Cultural Complex, Seminar Room A/B

Supported by JSPS KAKENHI Grant Number 16H02145, 17H02843, 19K03505

**Schedule**

| | 5 (Wed) B | 6 (Thu) A | 7 (Fri) B | 8 (Sat) A |
|---|---|---|---|---|
| 9:20-10:20 | | Matsuda | Sano | McShane |
| 10:50-11:50 | | Boileau | De Renzi | Sakuma |
| 13:00-14:00 | Miyachi | | (Lunch) | |
| 14:20-15:20 | Karuo | 14:00-15:00 | Nozaki | |
| 15:40-16:40 | Baba | 15:30-16:30 | Shimizu | |

**Abstract**

### Shinpei Baba (Osaka University)

**Title.** Neck-pinching of $\mathbb{C}P^1$-structures in the character variety

**Abstract.** A $\mathbb{C}P^1$-structure on a surface is a locally homogeneous structure modelled on $\mathbb{C}P^1$, and its holonomy representation is a homomorphism from the fundamental group of the base surface into $\mathrm{PSL}(2,\mathbb{C})$.

We consider a path of $\mathbb{C}P^1$-structures on a closed surface which diverges to infinity in the deformation space and yet its holonomy converges in the $\mathrm{PSL}(2,\mathbb{C})$-character variety. It is known that, along such a path, the conformal structure of the $\mathbb{C}P^1$-structure also diverges to infinity in the Teichmüller space. In this talk, we give certain geometric limits of the path under an addition assumption that the conformal structure is pinched along a loop.

### Michel Boileau (Aix-Marseille Université)

**Title.** Strongly quasipositive links, cyclic branched covers and L-spaces

**Abstract.** We are interested in the following question: which fibred strongly quasipositive links in $S^3$ admit a $n$-fold cyclic branched cover which is an L-space. When the Alexander polynomial is not a power of $(t-1)$ we show that the branching index must be $\leq 5$ and that the Alexander polynomial is a non-trivial product of cyclotomic polynomials. For a Birman-Ko-Lee positive closed braid with exponent $\geq 2$ we show that the link belongs to the restricted family of the so called simply laced arborescent links.

This is a joint work with Steve Boyer (University of Quebec at Montreal) and Cameron McA. Gordon (University of Texas at Austin).

**Marco De Renzi (Waseda University)**

**Title.** Non-semisimple TQFTs and quantum groups

**Abstract.** In recent years, non-semisimple constructions have substantially generalized the standard approach of Witten, Reshetikhin, and Turaev to quantum topology. Their appearance has led to powerful topological invariants and TQFTs with remarkable properties. In this talk, we will focus on two of these theories, based on two different flavors of the same algebraic ingredient: quantum groups. On one hand, the work of Costantino, Geer, and Patureau yields a family of invariants of closed 3-manifolds equipped with a cohomology class, called CGP invariants, which extend to graded TQFTs. These invariants are quite refined, as they contain the abelian Reidemeister torsion, but their definition involves a few technical aspects. More recently, together with Geer and Patureau, we defined a family of invariants of closed 3-manifolds, obtained by renormalizing a construction due to Hennings, which we managed to extend to TQFTs. Our approach avoids many of the technicalities of the CGP theory. We will show renormalized Hennings invariants coincide with CGP invariants associated with the zero cohomology class.

**Hiroaki Karuo (RIMS, Kyoto University)**

**Title.** The reduced Dijkgraaf–Witten invariant of twist knots in the Bloch group of $\mathbb{F}_p$

**Abstract.** For a closed 3-manifold $M$, a finite group $G$, and a representation $\pi_1(M) \to G$, (an invariant which is equivalent to) the image of the fundamental class of $M$ by a map $H_3(M) \to H_3(G)$ induced by the representation and a 3-cocycle of $G$ is called the Dijkgraaf–Witten invariant. In the case that $G = \mathrm{SL}_2\mathbb{C}$, Neumann described the Dijkgraaf–Witten invariant by using the Bloch group of $\mathbb{C}$ in 2004.

In this talk, in the case that $G = \mathrm{SL}_2\mathbb{F}_p$ ($\mathbb{F}_p$ denotes a finite field of prime order), I describe the Dijkgraaf–Witten invariant of the complements of twist knots by using the Bloch group of $\mathbb{F}_p$.

**Yoshifumi Matsuda (Aoyama Gakuin University)**

**Title.** On sepak takraw link

**Abstract.** A ball of Sepak takraw is similar to a soccer ball, but it consists of annuli. Replacing annuli with circles, we obtain an alternating link with rich symmetry, which we call sepak takraw link. Sepak takraw link can be obtained from a set of great circles in 2-sphere without triple crossings by replacing each intersection of two great circles with a crossing. We call such an alternating link a takraw link. Hopf link and Borromean rings are examples of takraw links. We study properties of sepak takraw link and takraw links including a characterization of sepak takraw link among takraw links.

**Greg McShane (Institut Fourier, Université Grenoble Alpes)**

**Title.** Angles, 1-coboundaries and identities

**Abstract.** In my thesis I proved an identity for lengths of closed simple geodesics on the hyperbolic once punctured torus by analysing the geometry of the Birman-Series set. Subsequently Bowditch gave an alternative proof using what he called Markoff maps. Many authors have extended Bowditch's method including M. Sakuma and S.P Tan.

We will relate Bowditch's method to functions defined on the Bass-Serre tree for the modular group $\mathrm{PSL}(2,\mathbb{Z})$. We will discuss why viewing these as coboundaries leads to a (formal) approach, via integration on Teichmueller space, to identities for lengths of closed simple geodesics on the hyperbolic one holed torus. We will relate this to the work of Hu, Tan and Zhang.

**Hideki Miyachi (Kanazawa University)**

**Title.** Complex analysis with Thurston theory in the Teichmüller theory

**Abstract.** In this talk, we will discuss a recent progress on unifying the complex analytical aspect and the topological aspect in the Teichmüller theory. We will give the Poisson integral formula (in Demailly's sense) for pluriharmonic functions on Teichmüller space which are continuous on the Bers closure. In our unifying procedure, we attempt to realize holomorphic functions on Teichmüller space as measurable functions on the space of projective measured foliations. In fact, we expect these functions are related via the Poisson integral formula. If time permits, we will deal with applications of our integral formula.

**Yuta Nozaki (Meiji University)**

**Title** Finiteness of the image of the Reidemeister torsion of a splice

**Abstract.** We regard the $SL(2,\mathbb{C})$-Reidemeister torsion of a 3-manifold $M$ as a $\mathbb{C}$-valued function on the character variety of $M$ and consider the image $RT(M)$ of this function. The set $RT(M)$ is known to be infinite when $M$ is the complement of the figure-eight knot or its double. In contrast, we prove that $RT(M)$ is a finite set if $M$ is the splice of two certain knots in $S^3$. The proof is based on an observation on the character varieties and $A$-polynomials of knots. This is a joint work with Teruaki Kitano.

**Makoto Sakuma (Hiroshima University)**

**Title.** Kleinian groups generated by two parabolic transformations

**Abstract.** I will give a progress report on my project to give a complete proof to Agol's classification of Kleinian groups generated by two parabolic transformations. I would also like to present a conjectural picture of the shape of such Kleinian groups. This is a joint work with Hirotaka Akiyoshi, Gaven Martin, Ken'ichi Ohshika, John Parker, and Han Yoshida.

**Taketo Sano (University of Tokyo)**

**Title.** Divisibility of Lee's class and its relation with Rasmussen's invariant

**Abstract.** Lee homology (a variant of Khovanov homology) over $\mathbb{Q}$ possesses the "canonical generators" as its basis. The generators (Lee's classes) $[\alpha(D, o)]$ are constructed combinatorially from an oriented link diagram $D$, one for each alternative orientation $o$ on $D$. Let $R$ be an integral domain. There exists a family of link homology theory $\{H_c(-; R)\}_{c \in R}$, where Khovanov's theory corresponds to $c = 0$ and Lee's theory corresponds to $c = 2$. For each $c \in R \setminus 0$, Lee's classes $[\alpha(D, o)]$ can be defined as elements in $H_c(D; R)$, but when $c$ is not invertible then they do not form a basis; in fact they are divisible by $c$-powers. We define the $c$-divisibility $k_c(D)$ of $[\alpha(D, o)]$ with $o$ the given orientation of $D$. For any link $L$ and its diagram $D$, we prove that $\bar{s}_c(L) := 2k_c(D) + w(D) - r(D) + 1$ is a link invariant, where $w$ is the writhe, and $r$ is the number of Seifert circles. We pose the question whether $\bar{s}_c$ coincides with Rasmussen's $s$-invariant. There are several evidences that support the affirmative answer. For instance, $\bar{s}_c$ is a link concordance invariant, and the Milnor conjecture can be reproved using $\bar{s}_c$. Also for the special case $(R, c) = (\mathbb{Q}[h], h)$, our $\bar{s}_c$ actually coincides with $s$ as knot invariants.

**Tatsuro Shimizu (Kyoto University)**

**Title.** On the $\Theta$-invariant of 3-manifolds

**Abstract.** Chern-Simons perturbation theory gives a sequence of invariants of 3-manifolds with local systems. Each term of the invariants are with a degree. Bott-Cattaneo's Theta-invariant is a version of the degree 1 term of Chern-Simons perturbation theory. There is a gap in their construction of the invariant. In this talk I revisit and modify the construction to remove the gap. I then discuss the higher degree term of Bott-Cattaneo's invariants. This is partly based on joint work with A. Cattaneo.